BENHA UNIVERSITY

ECE 344

# MICROWAVE FUNDAMENTALS PART1-Lecture 5

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*Many Slides from: ECE 5317_6351 Microwave Engineering* ***Prof. David R. Jackson***

## Example 1

USE ADS to:

-draw magnitude voltage across the line $\beta l = 2\pi$ or $\ell = \lambda$

-draw magnitude current across the line

-draw impedance across the line

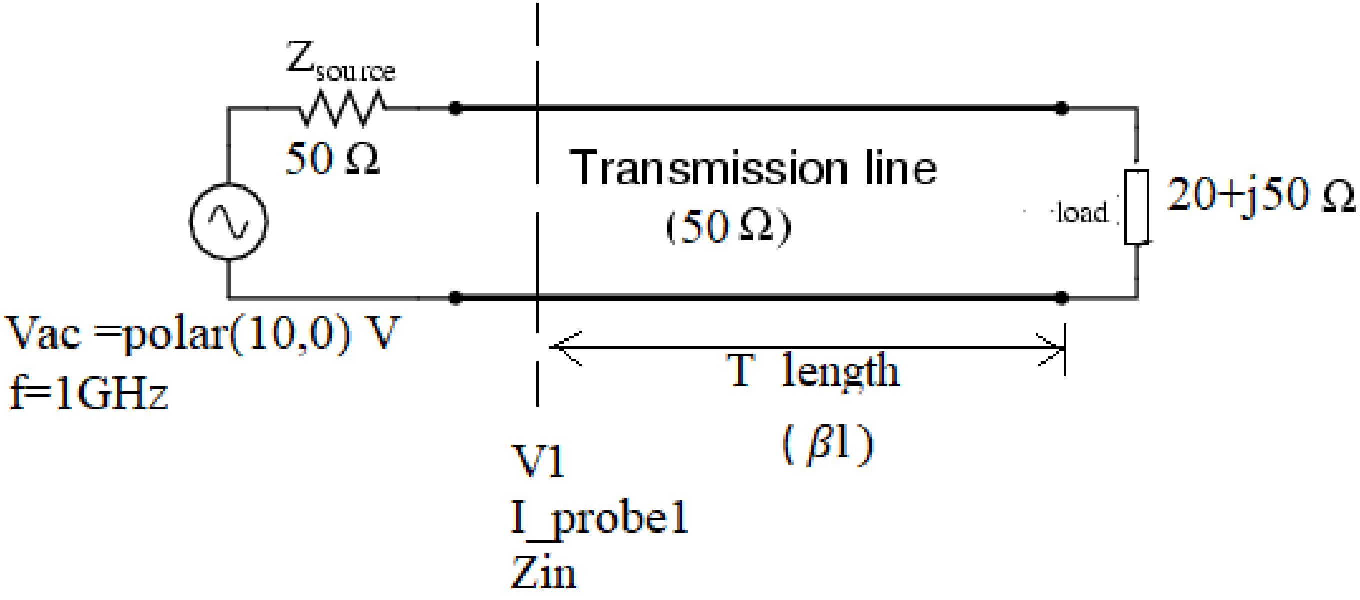
**observe mag(V),mag(I),Z every $\ell = \lambda/2$**

-Compute magnitude of voltage , current at load.

-verify input impedance at load from voltage/current Equals load impedance.

-find max voltage value and its position

-find min voltage value and its position

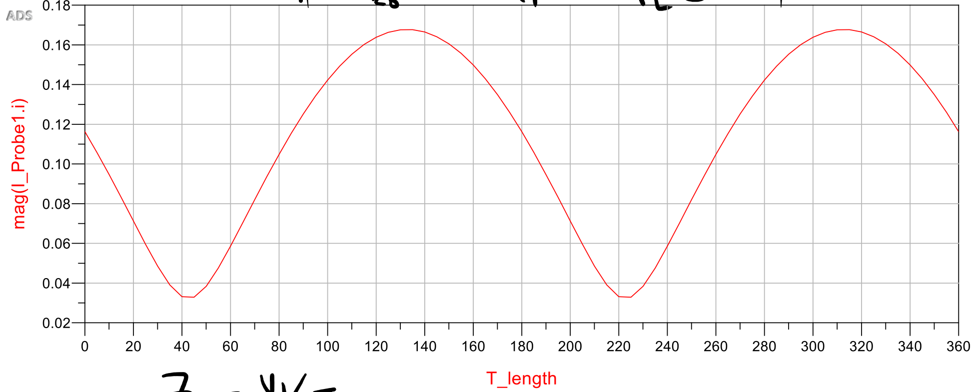


# Terminated transmission line repeats its voltage mag.,current mag. and impedance each $\lambda/2$

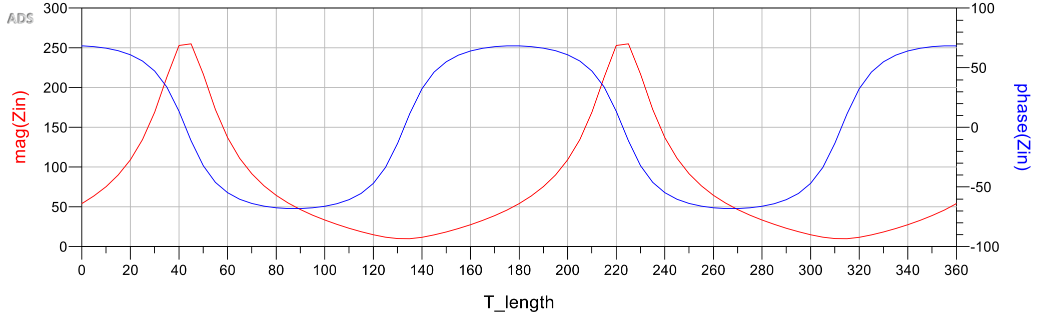
$$|V_1| = |V_0^+ e^{-j\beta z}(1 + \Gamma_L e^{2j\beta z})|$$

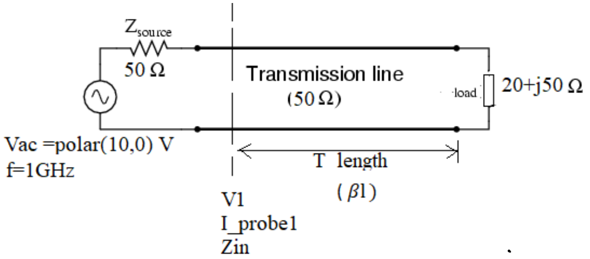
$\uparrow z=0$

$$|I_1| = \left|\frac{V_0^+}{Z_0} e^{-j\beta z}(1 - \Gamma_L e^{2j\beta z})\right|$$



$$Z_1 = V_1/I_1$$





$@\, z=0 \qquad V_0^+ = 5 \text{ Volt} \qquad Z_g = Z_0$

$V_1 = V_0^+(1 + \Gamma_L)$, $\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$

$V_1 = 5(1 + 0.678\angle 85.43)$

$V_1 = 6.26 \angle 32.67$

$I_1 = \frac{V_0^+}{Z_0}(1 - \Gamma_L) = 0.116 \angle -35.53$

$Z_1 = V_1/I_1 = 53.9 \angle 68.2$

$= Z_L$

$Z_L = 20 + j50 = 53.85 \angle 68.2$

$$\left.\begin{array}{l} V(\ell + \lambda/2) = -V(\ell) \\ I(\ell + \lambda/2) = -I(\ell) \end{array}\right\} Z_{in}(\ell + \tfrac{\lambda}{2}) = Z_{in}(\ell)$$

$V(\ell + \lambda) = V(\ell)$, $I(\ell + \lambda) = I(\ell)$

for $\beta l = 60^\circ$ $\quad V_0^+ = 5$ $\quad \Gamma_L = 0.678\angle 85.43$

Find $V(\beta l = 60)$, $I(\beta l = 60)$ ; $Z(\beta l = 60)$

Soln

$$V = V_0^+ e^{j\beta l}\left(1 + \Gamma_L e^{-2j\beta l}\right)$$

$$= 5\angle 60 \left(1 + 0.678\angle 85.43 \angle -120\right)$$

$$= 5.56 + j5.79 = 8\angle 46.13^\circ \text{ Volt}$$

$$I = \frac{V_0^+}{Z_0} e^{j\beta l}\left(1 - \Gamma_L e^{-2j\beta l}\right)$$

$$= \frac{5}{50}\angle 60 \left(1 - 0.678\angle 85.43 \angle -120\right)$$

$$= -0.011 + j\,0.057 = 0.0586\angle 101$$

$$Z = \frac{V}{I} = \frac{8\angle 46.13}{0.0586\angle 101} = 78.48 - j\,111.7 = 136.6\angle -54.9$$

$V_0^+ = 5$ Volt $\quad \Gamma_L = 0.678 \underline{|85.43}$

$$|V| = |V_0^+| \left| 1 + |\Gamma_L| e^{j\phi} e^{-2j\beta l} \right|$$

$$|V_{max}| = |V_0^+| (1 + |\Gamma_L|) = 5 \times 1.678 = 8.4 \text{ Volt}$$

$$|V_{min}| = |V_0^+| (1 - |\Gamma_L|) = 5 \times 0.322 = 1.6 \text{ Volt}$$

Max @ $\phi - 2\beta l = 0 \rightarrow \beta l = \dfrac{85.43}{2} = 42.7^\circ$

Min @ $\phi - 2\beta l = \pi \rightarrow \beta l = \dfrac{85.43 - 180}{2}$

$= -47.28 + \boxed{180}$

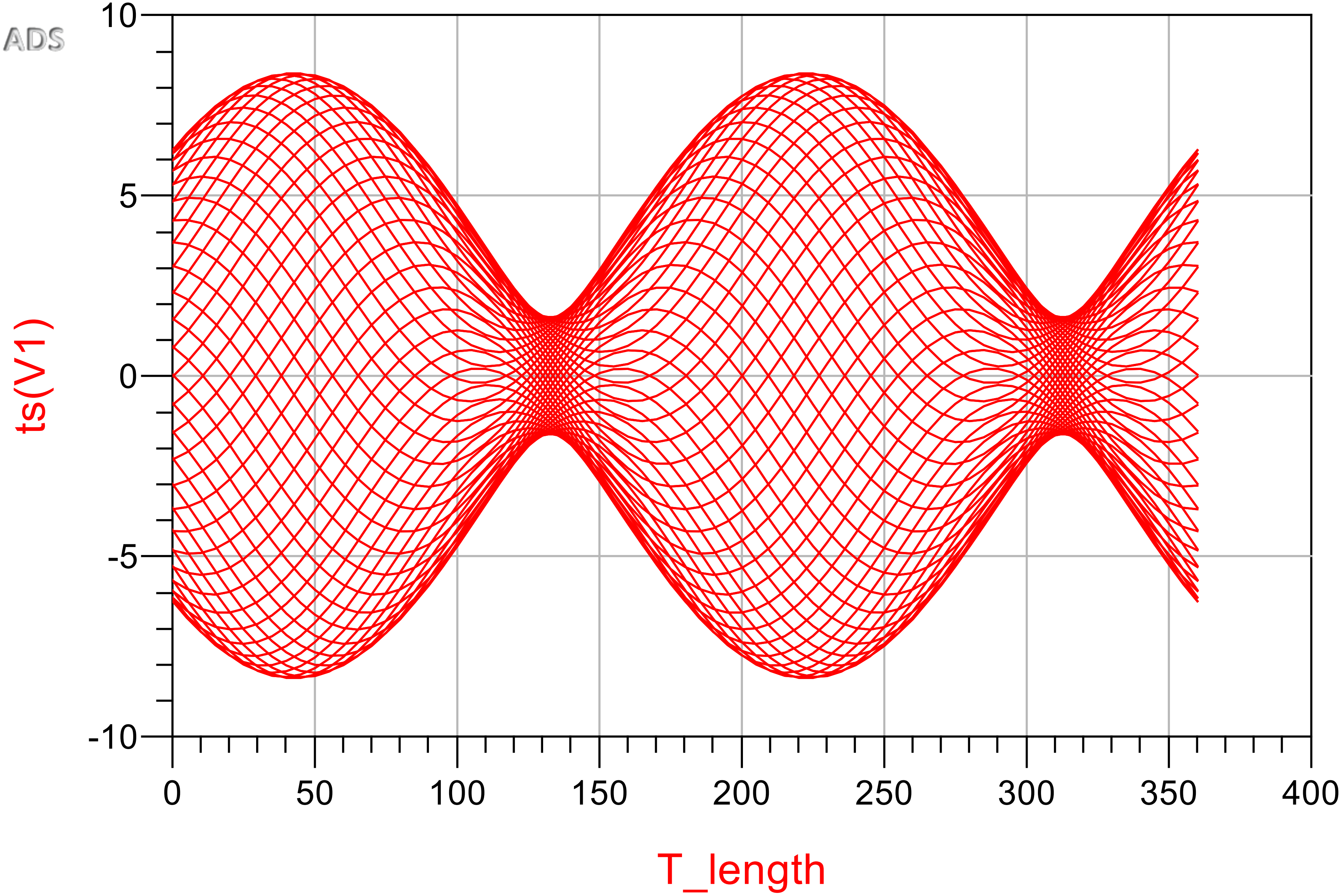
$= 132.7^\circ$

---

at $l = \lambda/2$ $\beta l = 180$

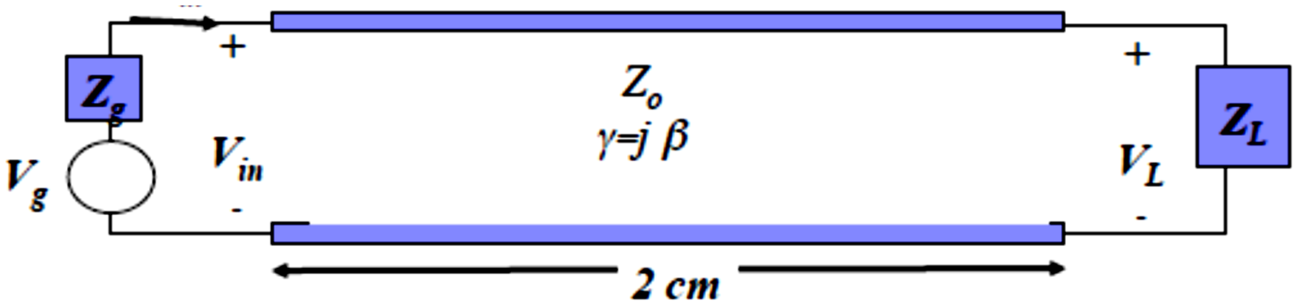
magnitude of Voltage Repeats every $l = \dfrac{\lambda}{2}$

as $|V(z + \lambda/2)| = |V(z)|$ or $\left| 1 + |\Gamma_L| e^{j\phi} e^{-2j\beta l} e^{-2j \times 180} \right| = \left| 1 + |\Gamma_L| e^{j\phi} e^{-2j\beta l} \right|$


ADS
10
5
0
-5
-10
ts(V1)
0
50
100
150
200
250
300
350
400
T_length

## Example 2

A 2cm lossless TL has $V_g$=10 volt, $Z_g$=60 Ω, $Z_L$=100+j80 Ω and $Z_o$=40 Ω, λ=10cm
Find the input impedance $Z_{in}$ and $V_{in}$

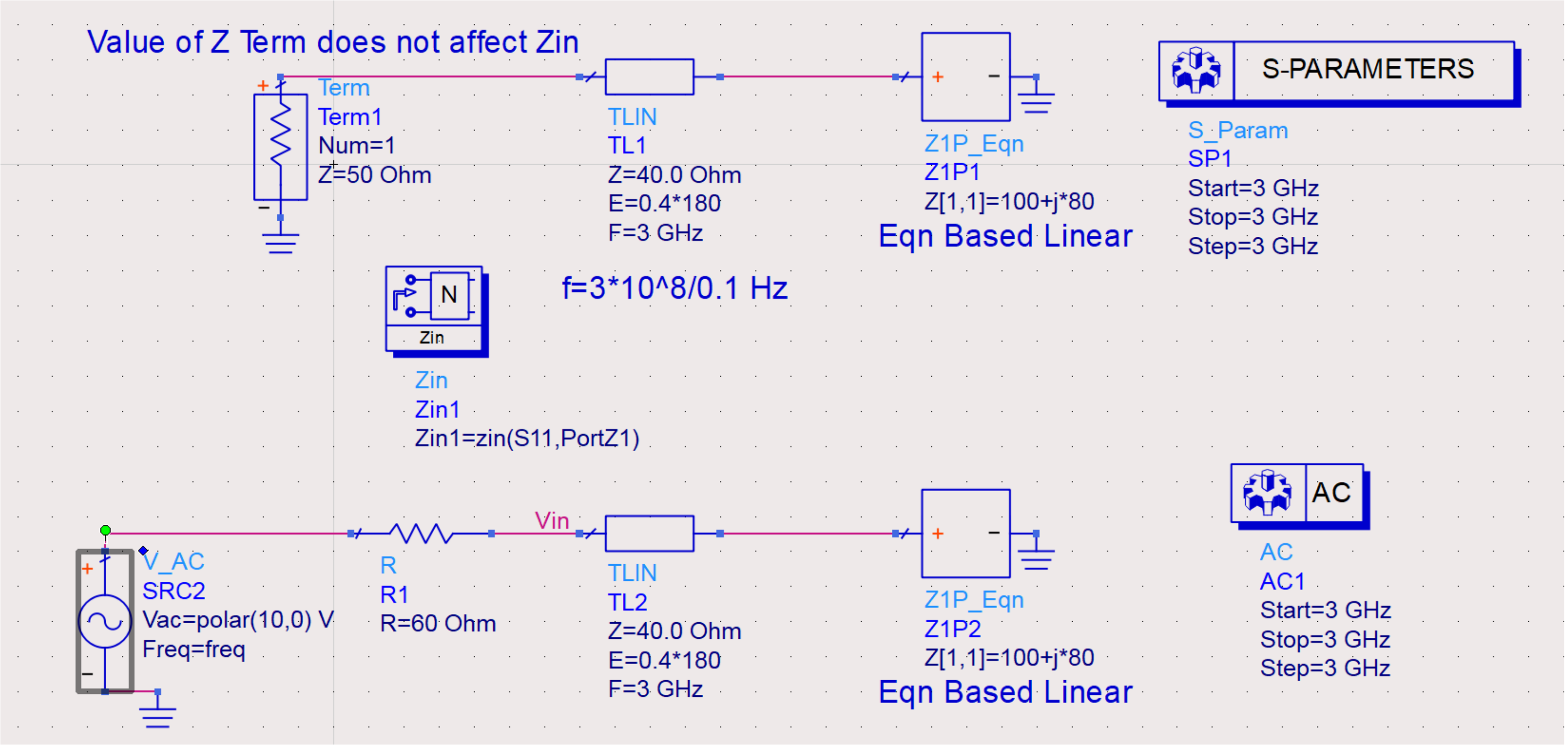


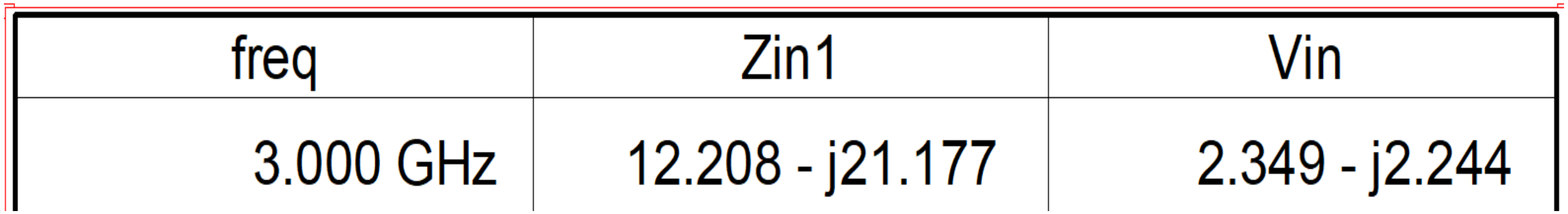
$$\mathrm{Z_{in}} = \mathrm{Z_0}\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \rightarrow \mathrm{Z_{in}} = 12.2 - j21.175$$

$$\mathrm{V_{in}} = \mathrm{V_g}\frac{\mathrm{Z_{in}}}{\mathrm{Z_{in}} + \mathrm{Z_g}} \rightarrow \mathrm{V_{in}} = 2.35 - j2.24$$

-compute incident voltage at load. *(Ans: 3.75∟-77.75)*

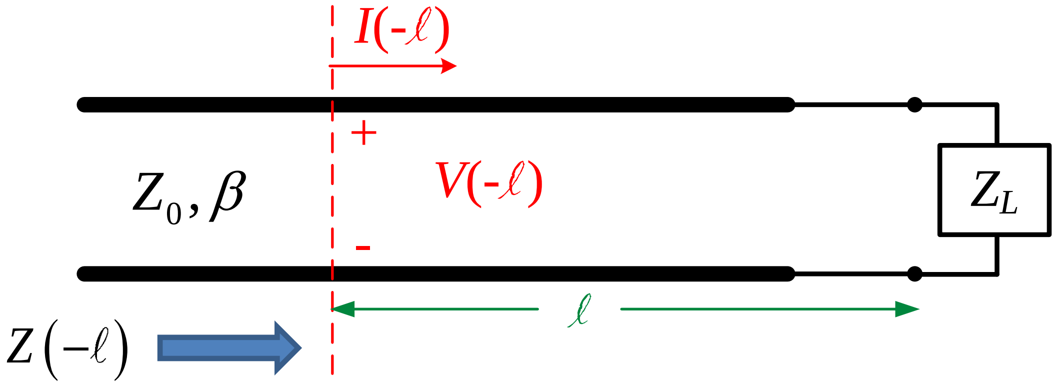
# Example 2 with ADS





| freq | Zin1 | Vin |
|---|---|---|
| 3.000 GHz | 12.208 - j21.177 | 2.349 - j2.244 |

# Matched Load



Ⓐ Matched load: ($Z_L = Z_0$)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = 0$$

No reflection from the load

$$\Rightarrow V(-\ell) = V_0^+ e^{+j\beta\ell}$$

$$I(-\ell) = \frac{V_0^+}{Z_0} e^{+j\beta\ell}$$

$$\Rightarrow Z(-\ell) = Z_0$$

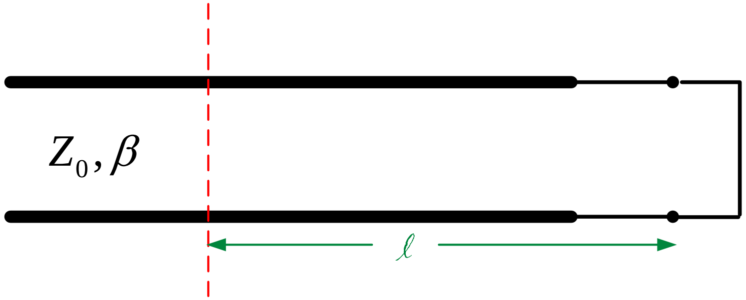
For any $\ell$

# Short-Circuit Load

Ⓑ Short circuit load: ($Z_L = 0$)

$$\Gamma_L = \frac{0 - Z_0}{0 + Z_0} = -1$$

$$\Rightarrow Z(-\ell) = jZ_0 \tan(\beta\ell)$$

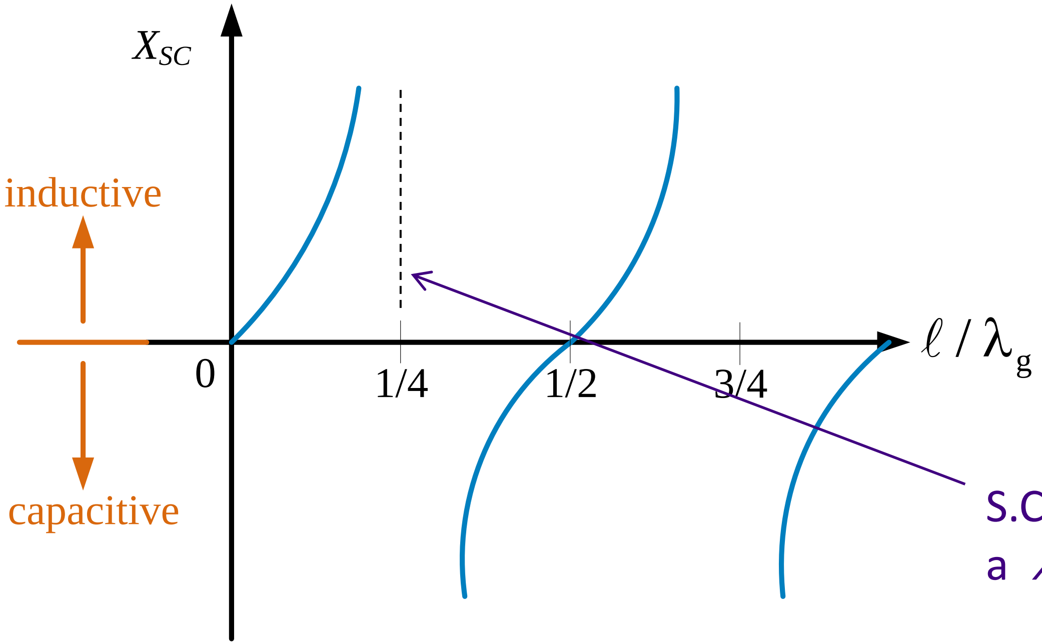


Note: $\beta\ell = 2\pi \frac{\ell}{\lambda_g}$

Always imaginary!

$$\Rightarrow Z(-\ell) = jX_{sc}$$

$$X_{sc} = Z_0 \tan(\beta\ell)$$



S.C. can become an O.C. with a $\lambda_g/4$ trans. line

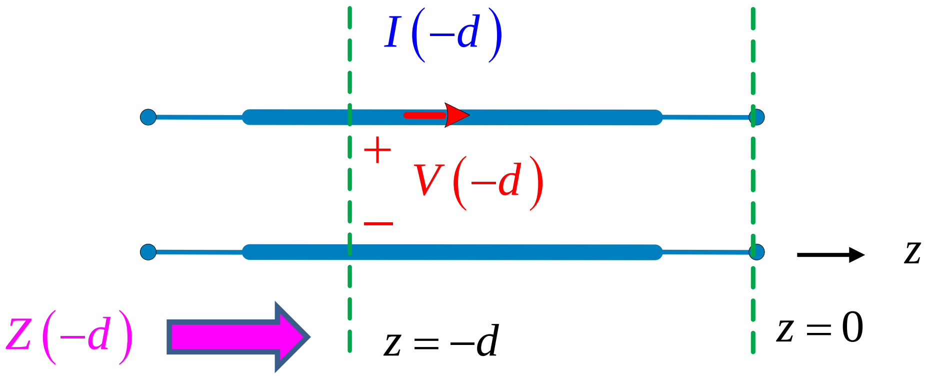
# Open-Circuit Load ($Z_L=\infty$)

ⓒ Open circuit load: (ZL = ∞)

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_L = \frac{\infty - Z_0}{\infty + Z_0}$$

$$\Gamma_L = +1$$



$$Z(-d) = Z_0\left(\frac{Z_L + jZ_0\tan(\beta d)}{Z_0 + jZ_L\tan(\beta d)}\right)$$

or

$$Z(-d) = Z_0\left(\frac{1 + j\left(\cancel{Z_0/Z_L}\right)\tan(\beta d)}{\left(\cancel{Z_0/Z_L}\right) + j\tan(\beta d)}\right)$$

$$Z(-d) = -jZ_0\cot(\beta d)$$

Always imaginary!

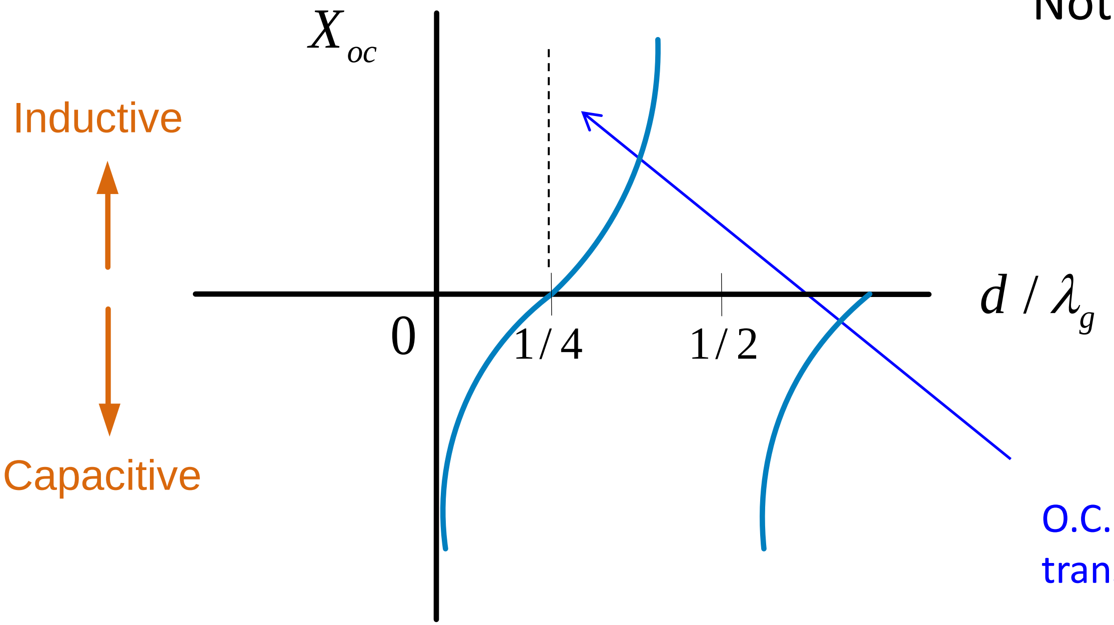
# Open-Circuit Load ($Z_L=\infty$)

$$Z(-d) = -jZ_0 \cot(\beta d)$$

$$Z(-d) = jX_{oc}$$

$$X_{oc} = -Z_0 \cot(\beta d)$$

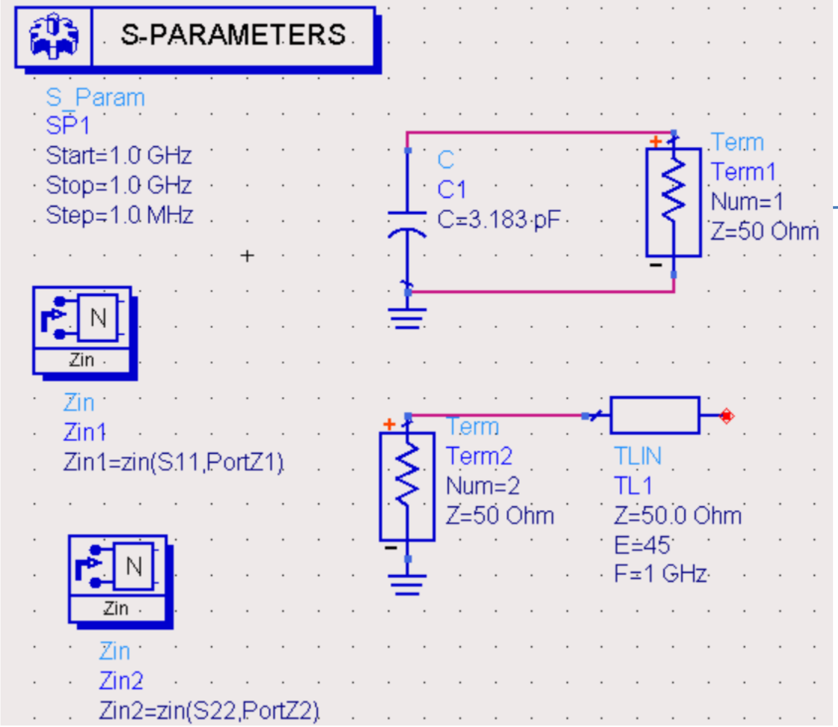
Note: $\beta d = 2\pi \dfrac{d}{\lambda_g}$

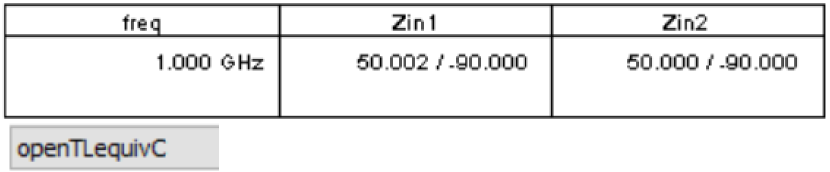


O.C. can become a S.C. with a $\lambda_g/4$ transmission line.

openTLequivC

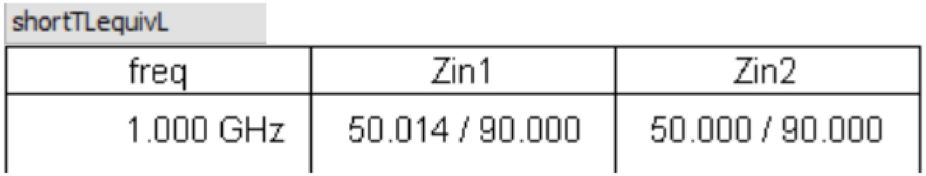
# Example 3 Open end and short end TL equivalent elements





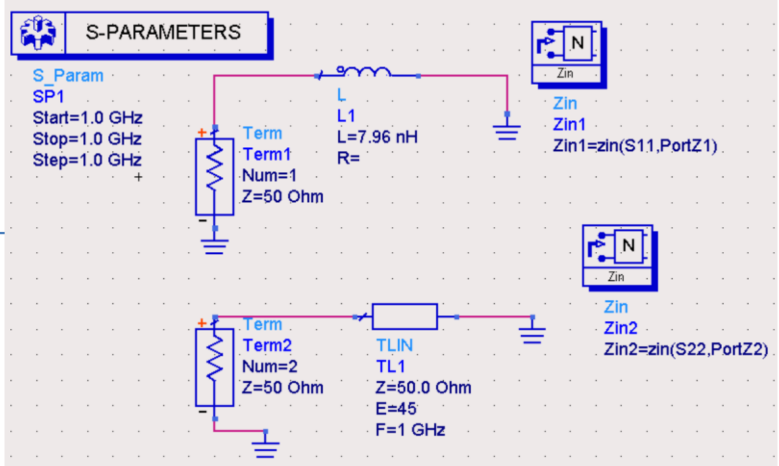
| freq | Zin1 | Zin2 |
|---|---|---|
| 1.000 GHz | 50.002 / -90.000 | 50.000 / -90.000 |

openTLequivC

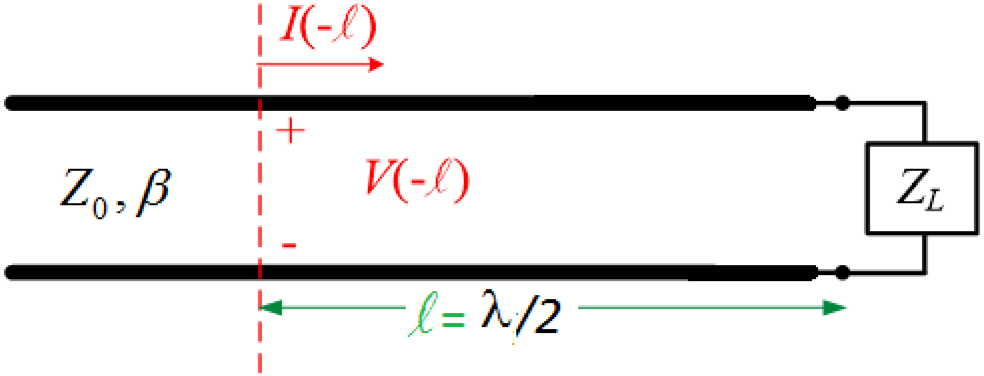


shortTLequivL

| freq | Zin1 | Zin2 |
|---|---|---|
| 1.000 GHz | 50.014 / 90.000 | 50.000 / 90.000 |



# Input impedance for a T.L. of length $\ell=\lambda/2$

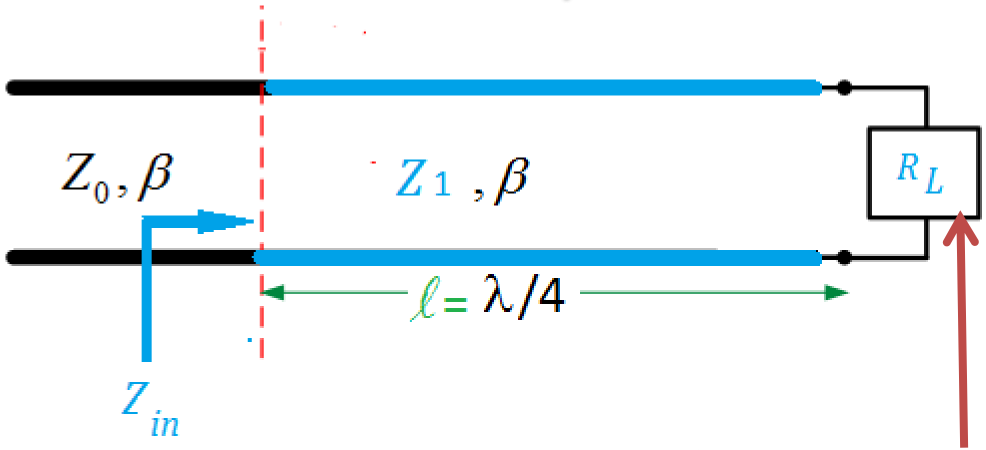


$$Z_{\text{in}} = Z_0 \frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}$$

$$\text{at } l = \lambda/2 \quad \Rightarrow \tan \beta l = 0 \rightarrow Z_{\text{in}} = Z_L$$

So any line( no matter its characteristic impedance) of length λ/2 or multiple of λ/2 , will look to (or have $Z_{in}$ ) $Z_L$ directly; (as if T.L. does not exist, i.e. T.L. does not transform $Z_L$ at its input)

# Input impedance for a T.L. of length ℓ=λ/4 (quarter wave transformer)



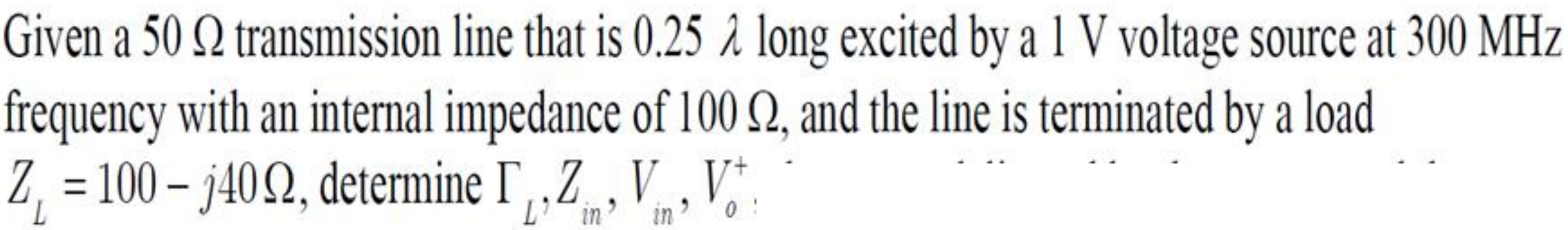
match pure resistive load impedance

$$Z_{\text{in}} = Z_1 \frac{Z_L \cos \beta l + jZ_1 \sin \beta l}{Z_1 \cos \beta l + jZ_L \sin \beta l},$$

$$\text{at } l = \lambda / 4, \beta l = \pi / 2 \rightarrow Z_{\text{in}} = \frac{Z_1{}^2}{Z_L} = Z_0$$

$Z_1 = \sqrt{Z_L Z_0}$ so input impedance at input of transformer look as $Z_0$

## Example 4



Given a 50 Ω transmission line that is 0.25 λ long excited by a 1 V voltage source at 300 MHz frequency with an internal impedance of 100 Ω, and the line is terminated by a load $Z_L = 100 - j40\,\Omega$, determine $\Gamma_L, Z_{in}, V_{in}, V_o^+$

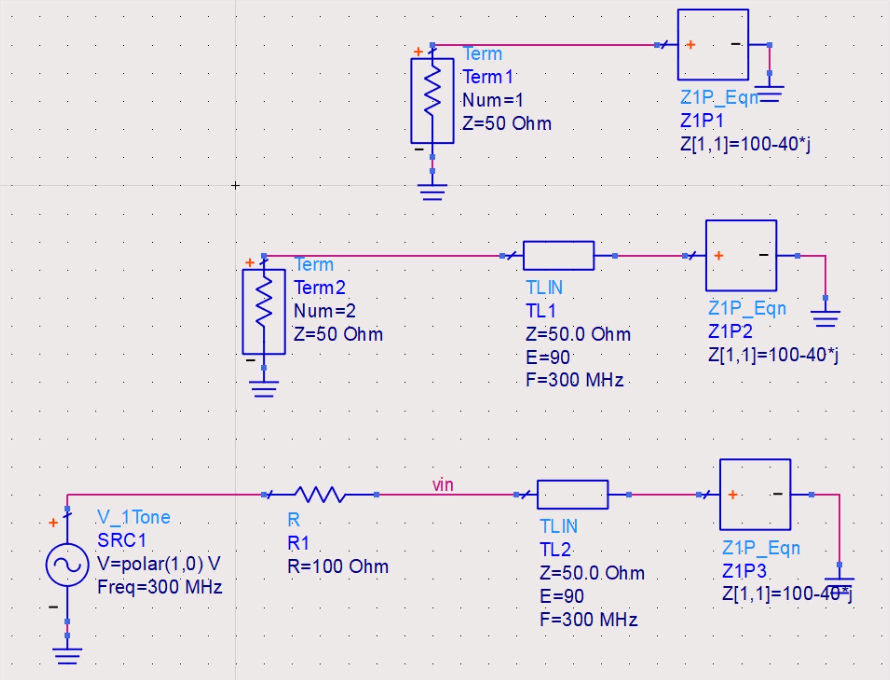
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = 0.378 - j0.166$$

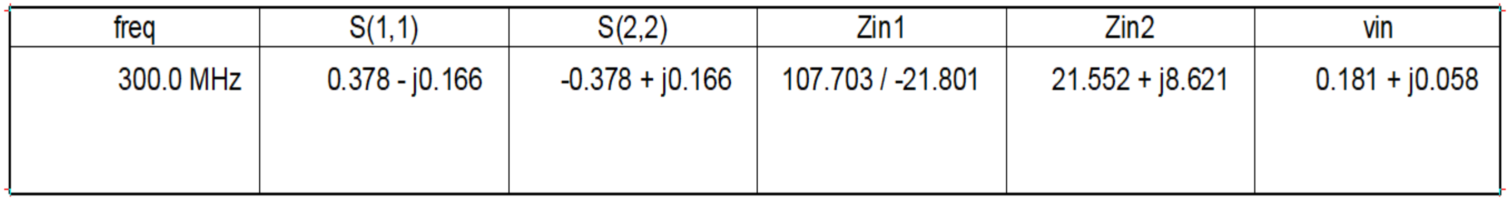
$$Z_{in} = Z_o * Z_o / Z_L = 21.55 + j8.62$$

$$V_{in} = V_{TH} \frac{Z_{in}}{Z_{in} + Z_{TH}} = 0.1814 + j0.058$$

$$V_o^+ = \frac{V_{in}}{e^{j\beta l}(1 + \Gamma_L e^{-2j\beta l})} = 0.0144 - j0.295$$

# Verify by ADS





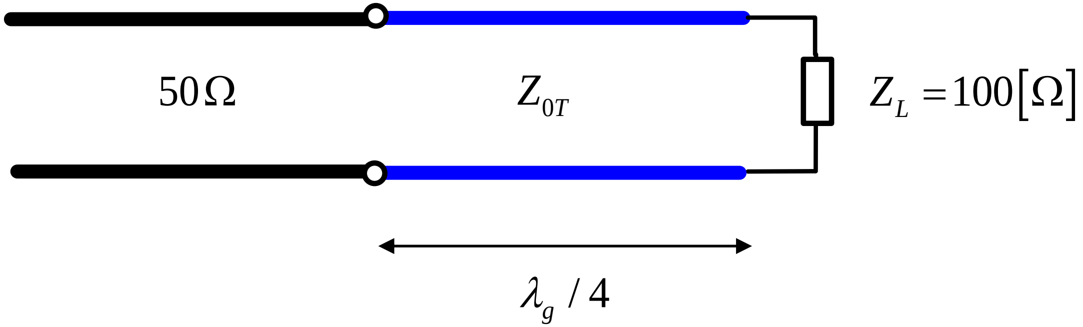
| freq | S(1,1) | S(2,2) | Zin1 | Zin2 | vin |
|---|---|---|---|---|---|
| 300.0 MHz | 0.378 - j0.166 | -0.378 + j0.166 | 107.703 / -21.801 | 21.552 + j8.621 | 0.181 + j0.058 |

## Example 5

Match a 100 Ω load to a 50 Ω transmission Line at a given frequency.

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{k} = \frac{2\pi}{k_0\sqrt{\varepsilon_r}} = \frac{\lambda_0}{\sqrt{\varepsilon_r}}$$

$$\lambda_0 = \frac{c}{f}$$



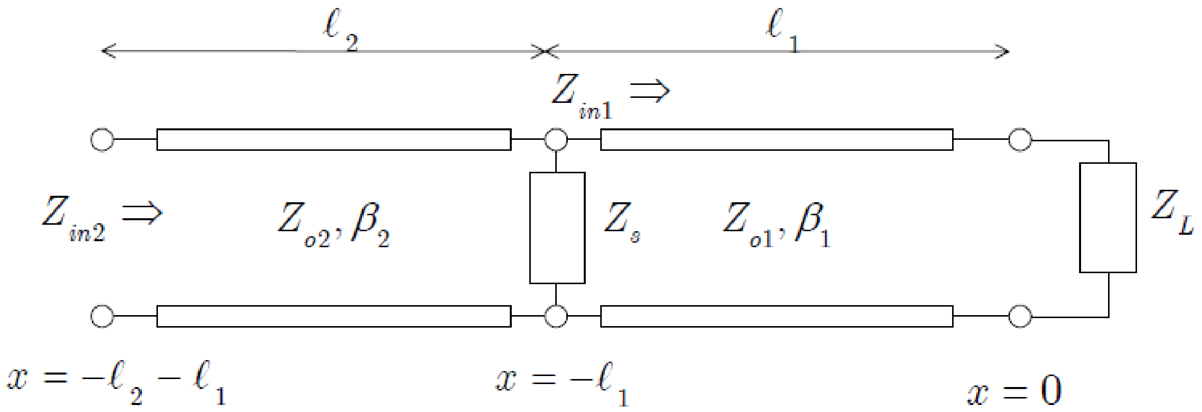
$$Z_{0T} = \sqrt{100 \times 50}$$

$$= 70.7$$

$$Z_{0T} = 70.7\,\Omega$$

# Shunt Loads

## A. Parallel Loads



*Determine* $Z_{in2} = Z_2(-\ell_1 - \ell_2)$:

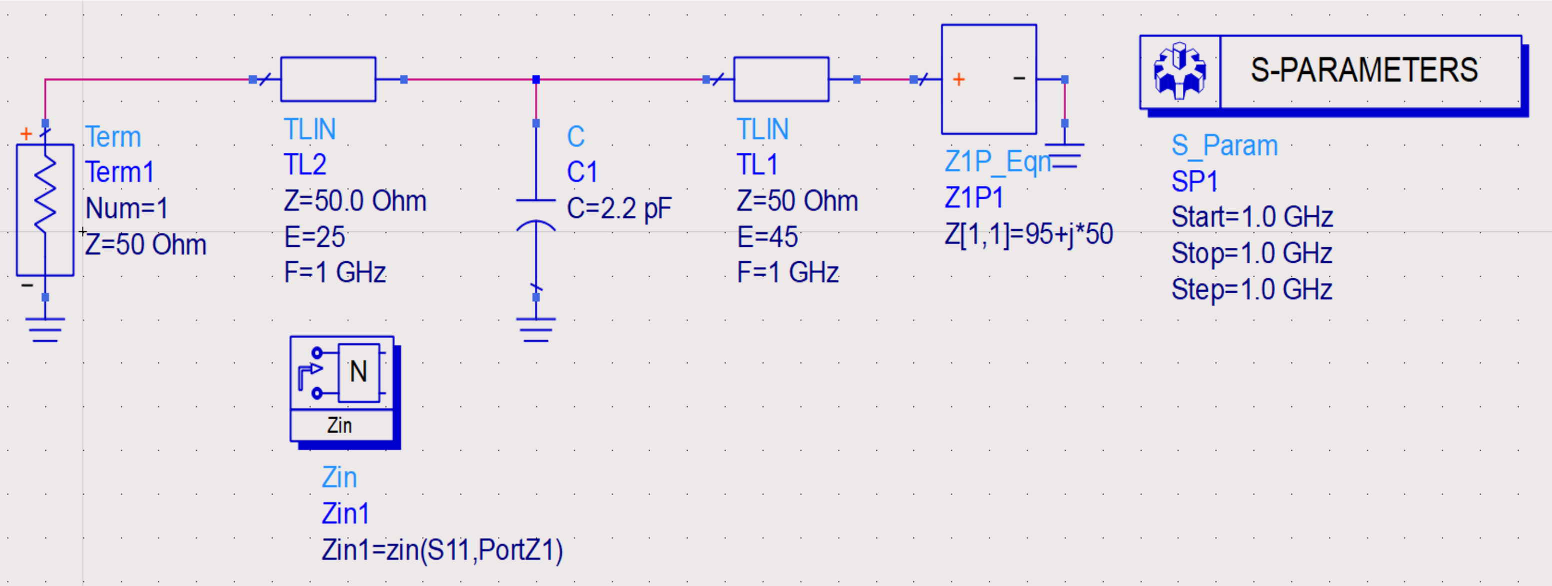
Solution Procedure:

1) Apply impedance match at $x=0$
2) Determine $Z_{in1}$
3) combine $Z_{in1}$ with $Z_s$ (How do we do this?)
4) Determine $Z_{in2}$

*Solution:*

$$Z_{in1} = Z_o \frac{Z_L + jZ_o \tan(\beta\ell_1)}{Z_o + jZ_L \tan(\beta\ell_1)}, \quad Z_{\|} = \frac{Z_s Z_{in1}}{Z_s + Z_{in1}}, \quad Z_{in2} = Z_o \frac{Z_{\|} + jZ_o \tan(\beta\ell_2)}{Z_o + jZ_{\|} \tan(\beta\ell_2)}$$

# Example 6



Term
Term1
Num=1
Z=50 Ohm

TLIN
TL2
Z=50.0 Ohm
E=25
F=1 GHz

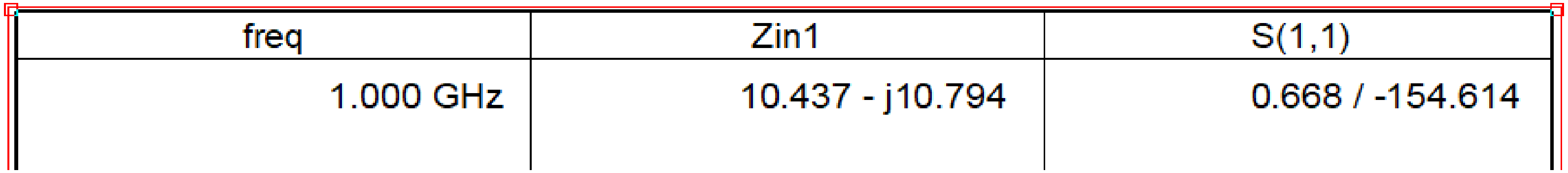
C
C1
C=2.2 pF

TLIN
TL1
Z=50 Ohm
E=45
F=1 GHz

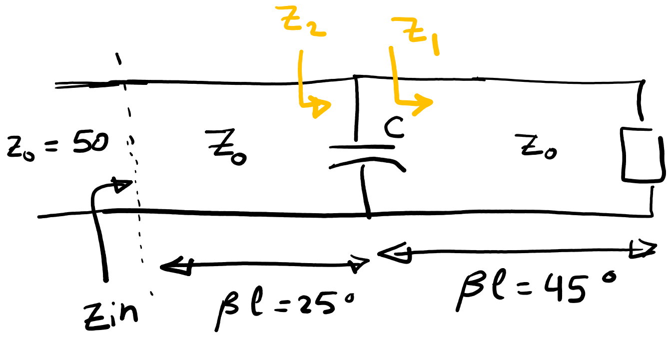
Z1P_Eqn
Z1P1
Z[1,1]=95+j*50

S-PARAMETERS
S_Param
SP1
Start=1.0 GHz
Stop=1.0 GHz
Step=1.0 GHz

Zin
Zin1
Zin1=zin(S11,PortZ1)



| freq | Zin1 | S(1,1) |
|---|---|---|
| 1.000 GHz | 10.437 - j10.794 | 0.668 / -154.614 |



$C = 2.2\,pF$

$f = 1\,GHz$

## Solution

$$Z_1 = 50\,\frac{(95+j50)+j50\tan 45}{50+j(95+j50)\tan 45}$$

$$Z_1 = 52.6 - j50 \longrightarrow Y_1 = 0.01 + j0.0094$$

$$J\omega C = j\,2\pi\times 10^9 \times 2.2\times 10^{-12} = j0.0138$$

$$Y_2 = Y_1 + J\omega C = 0.01 + j0.023 \Rightarrow Z_2 = 15.9 - j36.6$$

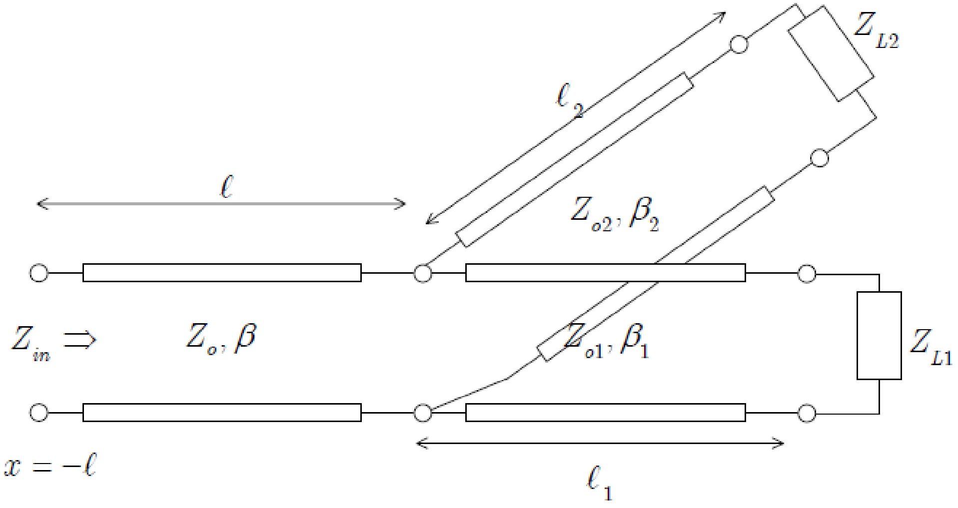
$$Z_{in} = 50\,\frac{(15.9-j36.6)+j50\tan 25}{50+j(15.9-j36.6)\tan 25} = 10.6 - j11$$

$$\approx 10.4 - j10.8$$

[Not Exact due to Approx. across solution]

$$\Gamma_{in} = \frac{Z_{in}-50}{Z_{in}+50} = -0.596 - j0.29 = 0.66\angle{-154}$$

## Parallel Lines:



*Determine* $Z_{in}$:

Solution Procedure:

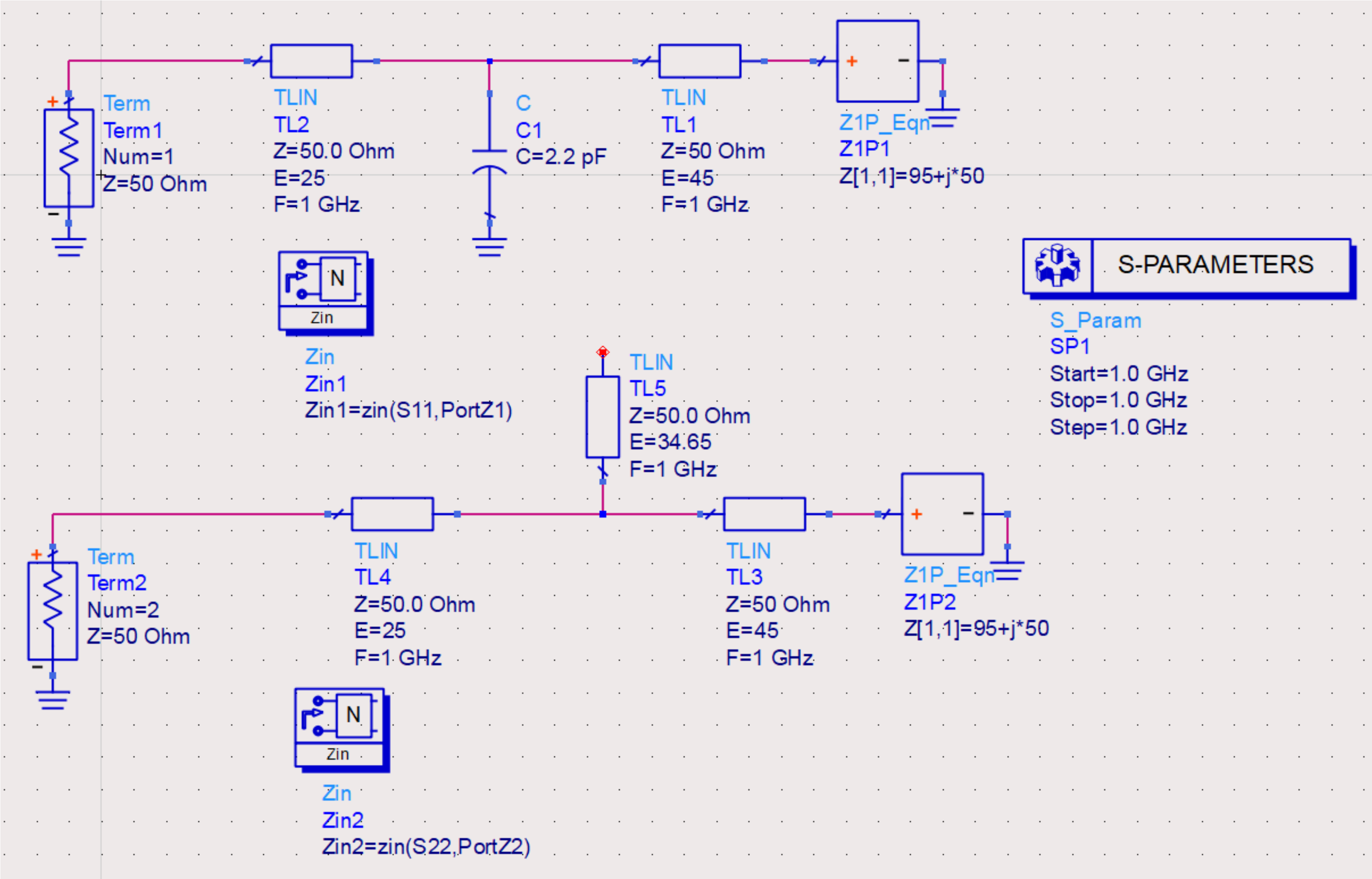
1) Determine $Z_{in}$ of lines 1 and 2
2) Determine effective load (how do they combine?)
3) Determine $Z_{in}$

*Solution:*

$$Z_{in1} = Z_{o1} \frac{Z_{L1} + jZ_{o1} \tan\left(\beta_1 \ell_1\right)}{Z_{o1} + jZ_{L1} \tan\left(\beta_1 \ell_1\right)}, \quad Z_{in2} = Z_{o2} \frac{Z_{L2} + jZ_{o2} \tan\left(\beta_2 \ell_2\right)}{Z_{o2} + jZ_{L2} \tan\left(\beta_2 \ell_2\right)}$$

$$Z_{\|} = \frac{Z_{in1} Z_{in2}}{Z_{in1} + Z_{in2}}, \qquad Z_{in} = Z_o \frac{Z_{\|} + jZ_o \tan\left(\beta \ell\right)}{Z_o + jZ_{\|} \tan\left(\beta \ell\right)}$$

# Example 6 with parallel o.c. TL instead of shunt C



Term
Term1
Num=1
Z=50 Ohm

TLIN
TL2
Z=50.0 Ohm
E=25
F=1 GHz

C
C1
C=2.2 pF

TLIN
TL1
Z=50 Ohm
E=45
F=1 GHz

Z1P_Eqn
Z1P1
Z[1,1]=95+j*50

S-PARAMETERS

S_Param
SP1
Start=1.0 GHz
Stop=1.0 GHz
Step=1.0 GHz

Zin
Zin1
Zin1=zin(S11,PortZ1)

TLIN
TL5
Z=50.0 Ohm
E=34.65
F=1 GHz

Term
Term2
Num=2
Z=50 Ohm

TLIN
TL4
Z=50.0 Ohm
E=25
F=1 GHz

TLIN
TL3
Z=50 Ohm
E=45
F=1 GHz

Z1P_Eqn
Z1P2
Z[1,1]=95+j*50

Zin
Zin2
Zin2=zin(S22,PortZ2)

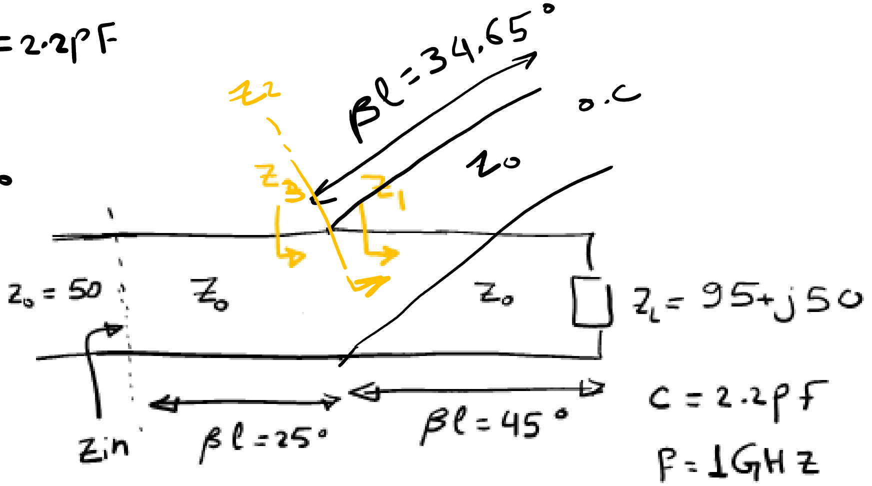
| freq | Zin1 | S(1,1) | Zin2 | S(2,2) |
|---|---|---|---|---|
| 1.000 GHz | 10.437 - j10.794 | 0.668 / -154.614 | 10.437 - j10.794 | 0.668 / -154.614 |

* Replace Shunt capacitor 2.2pF
withEquiv. o.c. T.L.

$$+J\frac{\tan\beta l}{Z_0} = J\omega C \rightarrow \tan\beta l = J\omega C Z_0$$

$$\beta l = \tan^{-1}(2\pi\times10^{9}\times2.2\times10^{-12}\times50)$$

$\underline{\text{Solution}} = 34.65°$



C = 2.2pF
f = 1GHz

$$Z_1 = 50\,\frac{(95+j50)+j50\tan45}{50+j(95+j50)\tan45}$$

$$Z_1 = 52.6-j50 \longrightarrow Y_1 = 0.01+j0.0094$$

$Z_{o.c} = -JZ_0\cot\beta l$ $\quad Y_2 = j\,\frac{\tan 34.65}{50} = j0.0138$ $\quad Y_1+Y_2 = 0.01+j0.0094+j0.0138 = 0.01+j0.023$

$Y_3 = Y_1 + Y_2$ $\quad = 0.01+j0.023 \Rightarrow Z_3 = 15.9-j36.6$

$$Z_{in} = 50\,\frac{(15.9-j36.6)+j50\tan25}{50+j(15.9-j36.6)\tan25} = 10.6-j11 \approx 10.4-j10.8$$

[Not Exact due to Approx. across solution]

$$\Gamma_{in} = \frac{Z_{in}-50}{Z_{in}+50} = -0.596-j0.29 = 0.66\angle{-154}$$